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Analysis of particle dispersion coefficient in solid-liquid fluidised beds

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Abstract

Solid liquid fluidised beds (SLFB) are frequently encountered in the mineral processing applications including ore leaching/washing and particle size classification based on difference

in settling velocity. Due to complex phase interactions, SLFB shows interesting characteristics such as phase segregation and mixing depending on variation in particle diameter, density difference and slip velocity. To characterise the multifaceted hydrodynamics of SLFB, often a diffusion-like parameter called dispersion coefficient is utilised. In literature, various empirical correlations of this parameter have been proposed however they are mostly system specific and lack a theoretical foundation. In this work, we report a model for dispersion coefficient along the line of definition for diffusion coefficient incorporating the mean free path of collision and interstitial fluid velocity as the characteristic velocity of collision. The model is tuned based on the available experimental data for mono as well as multi-particle system covering wide range of particle diameter (0.39 to 23 mm), liquid superficial velocity (0.0009 to 0.6 m s⁻¹) and Reynolds number (4 to 2820). To evaluate the capability of the proposed dispersion coefficient, mixing and segregation behaviour in binary SLFB system is simulated using a one dimensional convective-diffusive numerical model to predict the volume fraction distribution of each solid phase. Validation of the numerical model against available experimental data shows reasonably good agreement.

Keywords: Solid-liquid fluidised bed; binary particles; dispersion coefficient; mixing/segregation, convective-diffusive model.

1. Introduction

Due to excellent momentum, heat and mass transfer properties, solid-liquid fluidised beds (SLFB) are widely used in chemical process industries [1, 2] for particle classification, crystallization, ion exchange, chromatographic separation etc. These wide range of applications attract research attention comprising both experimental [3-

13] as well as numerical [13-28] investigation of the hydrodynamic behaviour of the SLFB system.

In a non-circulating SLFBs system, bed height expands with liquid superficial velocity below the elutriation limit. For a mono-component system, SLFBs expand almost in a homogeneous manner with a spatially and uniformly distributed manner resulting in uniform concentration of solids particles in both axial and radial directions. In a binary or multi-particle system, however, uniformity in solid concentration depends on the operating conditions. Generally, if a particle size distribution is involved, mean size of the particles decreases from bottom to top of the fluidised bed and might indicate partial or complete phase segregation. The degree of segregation occurs mainly due to the particle size and density ratio which diminishes as the superficial velocity is increased. In previous studies, the segregation and intermixing characteristics of SLFBs have been explained in terms of dispersion coefficient. Particle dispersion coefficient usually becomes insignificant when the bed is segregated, however, it increases due to the particle-particle and particle-fluid interactions with increasing superficial velocity which leads to an intermixed bed state.

It should be noted that the terminology ‘dispersion’ is relatively dissimilar from ‘diffusion’. Diffusion is a microscopic transport mechanism of mass, momentum and energy due to the random molecular movements caused by various driving forces such as concentration, velocity and temperature gradient. Dispersion on the other hand is a macroscopic phenomenon which is the combined exertion of advection and diffusion. In a multiphase reactor like SLFB, dispersion reflects mixing effects and is a measure of flow non-ideality. Dispersion behaviour in a system indicates deviation from the ideal plug flow behaviour and is often correlated to Peclet number (Pe) defined as the ratio of advection to diffusion. When Pe approaches zero, the flow is mostly diffusion

dominated and deviates significantly from the plug flow behaviour hence has dispersion coefficient approximately equal to zero. Conversely, when $Pe \rightarrow \infty$, dispersion dominates, and flow behaviour closely resembles plug flow characteristic.

Due to large length to diameter ratio, in SLFB system, it is more important to understand the axial dispersion coefficient rather than its radial counterpart as the degree of inter-mixing is described by the mixing of particles in the direction of fluid flow (vertical upward) [29]. A considerable literature review on the experimental and theoretical study indicates that dispersion coefficient depends on the system parameters such as fluid phase properties (density, viscosity, superficial velocity and liquid volume fraction) and solid phase properties (diameter, density, terminal settling velocity, minimum fluidisation velocity and volume fraction). Analogous to diffusion, from a theoretical perspective, dispersion characteristic of solids can be interpreted from the concept of the average distance between two successive particle collisions or mean free path [30]. Although several empirically derived correlations are available, we note that none of the previous studies attempted to describe dispersion coefficients from this perspective. Furthermore, a few of the available correlations can predict dispersion behaviour for a wide range of Reynolds number and large variation in the predicted value is not uncommon. Therefore, in line with this knowledge gap, it was thought desirable to develop a generalized correlation of axial solid dispersion coefficient from the definition of diffusion.

The paper is organised as follows: firstly a comprehensive review of the previous studies on axial particle dispersion coefficient and corresponding operating conditions to determine this parameter is presented in section 2; secondly, a generalised expression of the dispersion coefficient correlated over a wide operating range is presented in section 3; thirdly, in section 4, the proposed dispersion coefficient model is used for predicting the axial variation of solid

volume fraction in a binary SLFB system and mixing and segregation characteristics are analysed; and finally conclusion from this work is presented in section 5.

2. Previous work

Previous studies on dispersion coefficient in SLFBs can be broadly classified into three major categories – mono, binary and a multi-sized particle system. In this section, estimation of dispersion coefficient in each of these three categories is briefly reviewed.

2.1. *Dispersion in mono-sized particle system*

Muchi et al. [31] studied mixing of various mono-sized particles such as marble, fire brick and limestone in liquid fluidised beds where water was used as a fluidising medium. They determined the dispersion behaviour of the solid particles for the bed volume fraction, ranging from 0.59 to 0.82. Based on their experimental results, they proposed correlations for dispersion coefficient in both axial and radial locations of the SLFB. The dispersion coefficient was observed to increase with an increase in liquid superficial velocity hence liquid volume fraction.

Handly et al. [32] measured the dispersion coefficient of soda glass particles. They also reported increasing fluctuating velocity component of the particles i.e. the turbulence momentum transfer with an increase in the liquid volume fraction which was attributed to the higher values of particle dispersion coefficient. Carlos and Richardson [33] determined the dispersion of a glass particles fluidised in dimethyl phthalate medium. They used a particle detection technique and tracked the movement of a single tracer particle to evaluate the dispersion coefficient. They noted that the mixing process is mostly diffusional type hence the principal mechanism of the transport of particles can be considered to occur by turbulent dispersion at completely fluidized state.

Dorgelo et al. [34] measured the axial dispersion coefficient of particles in a mono-sized SLFB and proposed an empirical model as a function of the liquid superficial velocity based on

the random walk method. However, the dispersion coefficient correlation was found to be not equally dimensioned on both the left and right-hand sides. Using pressure drop measurement, Yutani et al. [35] developed a statistical method for estimating dispersion coefficient by determining variance of particle numbers in a fixed interrogation volume in a fluidised bed during the unsteady expansion phase. Dispersion behaviour of steady state fluidised particles was derived using a stochastic process (the so-called Ornstein-Uhlenbeck equation [36]), together with partition function of particle velocities.

Assuming homogeneity in the fluidised bed, Batchelor [37] first recognised the dispersion behaviour and made pioneering contributions to the transition analysis in fluidised bed systems. A theoretical correlation for particle dispersion coefficient was proposed as a function of particle phase diameter, liquid phase volume fraction and superficial velocity. It was further shown that dispersion behaviour is closely related to the stability of the fluidised beds. Kang et al. [38] investigated the effect of liquid superficial velocity, particle diameter and bed volume fraction on the dispersion of particles. Due to variation in liquid velocity and bed volume fraction, the fluctuating frequency and axial dispersion coefficient of particles shows their maxima (single peak) which also increased with an increase in particle diameter. On the other hand, with increasing particle size, liquid volume fraction at which dispersion coefficient exhibited a maximum, decreases. Based on experimental analysis, they also developed a model for particle dispersion coefficient.

Similar observation was also noted by Handly et al. [32] and Yutani et al. [35] while measuring turbulence intensity in a SLFB reactor. Joshi et al. [39] studied the stability analysis of solid-liquid fluidised bed reactors. They proposed that the particle dispersion coefficient can be expressed as a product of the turbulence length scale and the fluctuating velocity of the continuous phase theoretically. Based on the energy balance approach and the root means square velocity of the solid phase in the SLFB (experimental data obtained from Handley et al.

[32]), Joshi and Lali [40] developed an individual correlation for both the turbulence length scale and the fluctuating velocity component. The final expression of the dispersion coefficient [39] comprised the operating variables - particle diameter, volume fraction, and liquid superficial velocity.

2.2. *Dispersion in binary particle system*

Several researchers investigated mixing behaviour of different binary systems and quantified their observation in terms of dispersion coefficient. Kennedy and Bretton [11] investigated dispersion of glass particles in a binary SLFB considering simultaneous effects of both diffusion and convection. They extended the molecular diffusion model of Brotz [41] to calculate the dispersion coefficient and proposed another relationship for each of the particle types in a binary mixture. The developed dispersion model was used to determine the particle size distribution. The mean particle size gradient was observed to vary inversely with the expanded bed height, while the gradient was observed to remain independent of the bed volume fraction, liquid viscosity, particle density, and column diameter.

Juma and Richardson [10] investigated the mixing and segregation behaviour in binary solid mixtures fluidised by paraffin oil and observed that particle mixing increases with liquid superficial velocity. They employed combined diffusive and convective models to estimate dispersion coefficients in their system. For binary system of particles, the dispersion coefficient did not exhibit the usual maxima as noted in the mono-sized system. It was reported that the dispersion coefficient of the 2 mm solid was higher in the 2 and 4 mm binary mixtures as compared to the binary mixture of 2 and 3 mm solid particles. This is mainly because larger size (4 mm) solid particle produced a higher level of turbulence when compared with the smaller size (3 mm) particle in the SLFB.

van der Meer et al. [42] studied the mixing behaviour of ion exchange resin particles in water fluidised bed of four different diameters. They observed that the effect of column

diameter on particle dispersion is negligible which is because SLFB generally operated in a homogeneous regime where the relative velocity of solid and liquid generated turbulence in the system. They proposed a correlation for dispersion coefficient as a function of liquid superficial velocity ignoring the presence of another solid phase.

Based on a mass balance approach in presence of density gradients, Asif and Petersen [43] theoretically described the intermixing and segregation behaviour in binary SLFB systems. The mathematical model was used to evaluate concentration distribution and dispersion coefficients of the solid phases. By analysing experimental data reported by the other investigators [10, 12, 31, 33, 34, 38, 44], they proposed a unified correlation of dispersion coefficients based on appropriate dimensionless groups. However, the model over-predicted the experimental dispersion coefficient by an order of magnitude [45] at low particle Reynolds number. Using an electrochemical technique which detects collisions between tracer particles and probes.

Barghi et al. [4] estimated the dispersion coefficient of solid particles in a binary SLFB. They studied the effect of solid diameter, shape and density on the intermixing/segregation behaviour of a binary particle system. The dispersion coefficient was observed to increase with an increase in the liquid superficial velocity. It was also observed that the solid density had a much stronger effect on the segregation behaviour compared with particle diameter and shape. Bhattacharyya and Dutta [46] developed a computational approach for determining the axial concentration distribution of binary solid particles varying in diameter and/or density. They also proposed a correlation for the solid dispersion coefficient in terms of appropriate dimensionless parameters including their effective volume fraction. Bhattacharyya and Dutta [46] concluded that the proposed model can be used to predict the intermixing and segregation behaviour (axial concentration) of a binary system of particles without any fitting parameter.

Based on experimental data from sampling and pressure measurement technique, Galvin et al. [9] proposed a dispersion model for a binary mixture of particles as function of particle

diameter, solid volume fractions and interstitial fluidisation velocity. They reported that because of the change in the solid volume fraction, the axial dispersion coefficient varied significantly. By employing their own dispersion model, they developed an analytical approach to predict axial concentration of binary particles based on the approaches suggested by Kennedy and Bretton [11] and Asif and Petersen [43]. Although this mathematical approach has made a significant contribution to elaborate intermixing/segregation behaviours, the model is only valid for a steady state binary system. Additionally, it failed to capture the concentration gradient at the top section of the fluidised bed. Furthermore, generalisation of their dispersion model by performing validations with the other reported experimental data was not discussed.

2.3. *Dispersion in multi-particle system*

Al-Dibouni and Garside [47, 48] quantified the dispersion behaviour in the multi-particle system (diameter ranging between 1.8 and 3.0 mm) by using Kennedy and Bretton [11] model. These authors investigated the mixing and segregation behaviour and observed that dispersion coefficient reached a maximum value of $0.002 \text{ m}^2 \text{ s}^{-1}$ at a certain bed volume fraction of 0.7. This behaviour was attributed to the complete intermixing throughout the bed at particle size ratio of 2.0 whilst the reverse behaviour (complete segregation) ensued when the particle size ratio greater than 2.2.

Murli et al. [45] measured dispersion coefficient of a multi-sized particle system over a wide range of liquid superficial velocities and particle diameters. Based on pressure measurements in a solid-liquid fluidised bed and other experimental data reported in the previous literature [4, 10-12, 33, 42, 47], they proposed a generalised correlation for particle dispersion coefficient which covers a wide range of Reynolds number ($Re_\infty = 4$ to 1233). The dispersion coefficient was observed to increase with both liquid superficial velocity and particle diameter. Chavan and Joshi [5] used the pressure gradient method to analyse binary as well as multi particles mixing and segregation phenomena in a SLFB ignoring any wall friction and

acceleration effects in measuring the pressure gradient between two pressure taps on the column. They found that dispersion characteristics depended on the particle diameter ratio, liquid velocity, and the bed composition. They also reported that the dispersion coefficient increases with an increase in the liquid superficial velocity and particle size.

A summary of the theoretical models and experimental techniques to quantify particle dispersion coefficient in SLFB system reported in previous literature is given in Table 1. Available theoretical and empirical correlations for the dispersion coefficients available in the literature are provided in Table 2.

3. Model development

The literature review presented in Section 2 indicates dependency of dispersion coefficient on various system parameters such as fluid phase properties (density ρ , viscosity μ , superficial velocity V_L , liquid volume fraction ϵ_L), solid phase properties (diameter d_s , density ρ_p , volume fraction ϵ_L) and also system characteristics such as terminal settling velocity $V_{S\infty}$ and minimum fluidisation velocity V_{mf} . Therefore, the dispersion coefficient of the particle phase is expected to be a function of the following parameters:

$$D_s = f(d_s, \epsilon_L, V_L, V_{S\infty}, V_{mf}). \quad (13)$$

Drawing an analogy with the diffusion phenomenon, according to the kinetic theory, dispersion coefficient can be expressed as a product of mean free path (λ_s) and the fluctuating velocity component (V') of particles [30] as follows:

$$D_s \sim \lambda_s V' \quad (14)$$

In kinetic theory, mean free path is defined as the distance travelled by a molecule between two successive collisions. In a fluidised bed system, this quantity essentially depends on the solid volume fraction (to account for the interacting neighbouring particles) and particle diameter (travelled distance) and can be written as follows [49]:

$$\lambda_s = \frac{d_s}{6(1-\epsilon_L)}, \quad (15)$$

In a non-circulating fluidised bed, particles exhibit random fluctuating motion with zero average velocity. Admittedly, it is challenging to obtain an expression of the fluctuating velocity of particles which depends on both the complex particle-fluid, particle-particle and particle-solid wall interactions. For that reason, from the functional dependency presented in Eq. (13), this velocity term in the current modelling is treated rather simplistically.

Average fluid velocity in the system can be expressed as $(V_L - V_{mf})/\epsilon_L$ which denotes the

interstitial velocity of fluid. This parameter also represents the maximum possible velocity of particles if a no-slip condition is assumed with physical basis that at $V=V_{mf}$, dispersion coefficient is zero as particle velocity is zero. Particle terminal settling velocity, $V_{S\infty}$, and minimum fluidisation velocity, V_{mf} , constitute the range of operating fluid velocity and particle velocity would be lower than this range due to slip condition, i.e. particle velocity is zero at minimum fluidisation and slightly less than the fluid velocity at the terminal settling velocity. A ratio $(V_{S\infty}/V_{mf})$ is additionally incorporated as an effect of the operating conditions on the dispersion coefficient. A direct dependency of dispersion coefficient with this ratio could be readily realised. It could be theoretically shown that this ratio of the terminal settling velocity to minimum fluidisation velocity reduces to bed void fraction only for both laminar (Stokes regime) and turbulent flow (Newton regime) scenario. With this consideration, the final form of the dispersion coefficient can be written with a proportionality constant K as follows:

$$D_s = K\lambda_s \left(\frac{V_L - V_{mf}}{\epsilon_L} \right) \left(\frac{V_{S\infty}}{V_{mf}} \right), \quad (16)$$

Minimum fluidisation velocity V_{mf} in Eq.(16) can be calculated by the following expression proposed by Wen and Yu [50]:

$$V_{mf} = \sqrt{\left(33.7 \frac{\mu_L}{d_s \rho_L} \right)^2 + 0.0408 d_s g \left(\frac{\rho_s - \rho_L}{\rho_L} \right)} - 33.7 \frac{\mu_L}{d_s \rho_L}, \quad (17)$$

Several other correlations for estimating minimum fluidisation velocity were also examined nonetheless it was noted that Eq.(17) provides a better estimation of dispersion coefficient (Eq.16).

In the present numerical modelling, both the particle Reynolds number and terminal settling velocity were calculated using the approach reported in Lali et al. [53]. The terminal settling velocity of a solid particle was estimated as a function of Reynolds number as:

$$V_{\infty} = \frac{Re_s \mu_L \rho_L}{d_s}, \quad (18)$$

Particle Reynolds number in Eq. (18) for four different regimes was expressed in terms of Archimedes number Ar (in Lali et al.'s [53] work, it is however defined as Galileo number Ga) as:

$$\begin{cases} Re_s = Ar/18; & \text{for } Re_s < 1; Ar < 1.8 \\ Re_s = (Ar/18)^{0.8}; & \text{for } 1 < Re_s < 56; 1.8 < Ar < 2.6 \times 10^3 \\ Re_s = 0.45Ar^{0.61}; & \text{for } 56 < Re_s < 10^3; 2.6 \times 10^3 < Ar < 3.3 \times 10^5 \\ Re_s = 1.732Ar^{0.5}; & \text{for } Re_s > 10^3; Ar > 3.3 \times 10^5 \end{cases}, \quad (19)$$

where Archimedes number defined as the ratio of buoyancy force to viscous force was given as

$$Ar_i = \frac{d_{Si}^3 \rho_L (\rho_{Si} - \rho_L) g}{\mu_L^2}, \quad (20)$$

A wide range of experimental operating conditions involving Reynolds number ($4 \leq Re \leq 2820$) and bed void fraction ($0.43 \leq \epsilon_L \leq 0.95$) were used which comprised 205 dispersion coefficient data from the previous literature [4, 5, 10, 11, 31-35, 38, 42, 45, 47]. The value of K was estimated iteratively by minimising the overall root mean square deviation (RMSD) between the experimental data and the proposed model which was calculated as follows:

$$RMSD = \sqrt{\frac{\sum_{i=1}^N (D_{\text{exp}} - D_{\text{model}})^2}{N}}, \quad (21)$$

where D_{exp} is the experimentally determined dispersion coefficient, D_{model} is corresponding model prediction and N is the number of data points.

The final value of K was found to be 0.15 ensuring that the slope of the fitted diagonal line (of the form $y = Ax$) A is ~ 1.0 with a maximum possible R^2 value (~ 0.7). All experimental data can be noticed to fall on both sides of the diagonal line within a $\pm 30\%$ deviation band. The constant K is a non-dimensional quantity which makes Eq. (16) to be more generic in nature.

To compare the goodness of the other reported dispersion coefficient models, Table 3 presents the corresponding RMSD, slope of the linear fit (A) and R^2 value based on the same experimental dataset. It is apparent that other models produce relatively low R^2 values (<0.7) and less parity (deviation from the diagonal line slope $A = 1$) which ascertains the fact that the present model is indeed a better fit ($\text{RMSD} \sim 0.0006 \text{ m}^2 \text{ s}^{-1}$) compared to the existing correlations (see Fig. 1).

4. Convective-diffusive model for the binary system

A one dimensional convective-diffusive model (Galvin et al. [9], Kennedy and Bretton [11]) was used to describe the axial variation of solid species i ($i = 1, 2$) as follows:

$$D_{Si} \frac{\partial \epsilon_{Si}}{\partial z} = \epsilon_{Si} V_{\text{seg},i}, \quad (22)$$

where z is the axial distance, and ϵ_{Si} , D_i , are the solids volume fraction and dispersion coefficient for solid species i respectively; and $V_{\text{seg},i}$ is the respective segregation velocity which is given following Asif and Petersen [43] as:

$$V_{\text{seg},i} = (1 - \epsilon_{nSi}) V_L V_{1,2}, \quad (23)$$

where V_L is the liquid superficial velocity, and $V_{1,2}$ is the normalised segregation parameter for solid species $i = 1, 2$, which can be expressed as:

$$V_{1,2} = \left(1 - \frac{V_{S1\infty}}{V_{S2\infty}} \right). \quad (24)$$

In Eq. (24), $V_{S\infty}$ is the terminal velocities for solids species in an infinite quiescent medium which is computed from Eq. (18)-(20). In Eq. (23), ϵ_{nSi} is the corresponding normalised volume fraction of species i , and remains conserved [43], i.e.:

$$\sum_{i=1}^2 \epsilon_{nSi} = 1, \quad (25)$$

where

$$\epsilon_{nSi} = \frac{\epsilon_{Si}}{\epsilon_{mSi}}, \quad (26)$$

and ϵ_{mSi} is given by [51] as follows:

$$\epsilon_{mSi} = 1 - \left(\frac{V_L}{V_{Si\infty}} \right)^{1/n_i}, \quad (27)$$

where exponent n_i is given by Richardson and Zaki [52] as:

$$n_i = \begin{cases} \{4.35 + 19.5(d_{Si}/D_c)\} & \text{for } Re_{Si} < 0.2 \\ (Re_{Si})^{-0.03} \{4.35 + 17.5(d_{Si}/D_c)\}; & \text{for } 1 < Re_{Si} < 200 \\ (Re_{Si})^{-0.1} \{4.45 + 18(d_{Si}/D_c)\}; & \text{for } 200 < Re_{Si} < 500 \\ 4.45(Re_{Si})^{-0.1}; & \text{for } Re_{Si} > 500 \\ 2.39; & \end{cases} \quad (28)$$

where, Re_{Si} , is the particle Reynolds number, d_{Si} is the spherical diameters for solid species, i ; ρ_{Si} , μ_L are solid species densities and liquid dynamic viscosity, respectively, $V_{Si\infty}$ is particle terminal settling velocity computed from Eq. 18-20, and D_c is column diameter.

In order to solve Eq. (22), the axial dispersion coefficient D_{Si} needs to be quantified. Following the analysis presented in Section 3, the proposed dispersion coefficient in Eq. (16) was utilised:

$$D_{Si} = 0.15\lambda_{Si} \left(\frac{V_L - V_{mfi}}{\epsilon_L} \right) \left(\frac{V_{Si\infty}}{V_{mfi}} \right), \quad (29)$$

where ϵ_L is the liquid volume fraction and is given by the following conservation equation:

$$\epsilon_L = 1 - \sum_{i=1}^2 \epsilon_{Si}, \quad (30)$$

The above equation represents the well-known convective-diffusive framework reported in several previous studies [9, 11] to determine the solid volume fraction distribution in the axial direction. A widely used hypothetical notion in the binary SLFB systems is that total bed height is additive [2], i.e. bed height equals to the sum of heights of the two individual component

beds fluidised at the same liquid superficial velocity with insignificant deviation observed [8]. In the present work, solid 1 is the lower settling velocity solid phase (smaller diameter or less dense than the solid phase 2), which occupies the top portion of the fluidised bed and solid 2 (heavier solid species) occupies the bottom part of the fluidised bed. A small intermixing zone was observed in the experiments, height of which was much less ($\sim 5-10$ mm) compared with the individual component bed height. For this reason, it was justifiable to use Richardson-Zaki [52] correlation to determine the bottom bed height (solid phase 2) while the top bed height (solid phase 1) was determined based on mass balance.

Substituting the expressions for ϵ_L , D_{Si} and $V_{Seg,i}$ into Eq. (22) yields the following equation for the axial volume fraction profile for particle species i :

$$\frac{\partial \epsilon_{Si}}{\partial z} = \frac{\epsilon_{Si} \left(1 - \sum_{i=1}^2 \epsilon_{Si} \right) (1 - \epsilon_{nSi}) (V_{S2\infty} - V_{S1\infty}) V_L V_{mfi}}{0.15 \lambda_{Si} V_{Si\infty} V_{S2\infty} (V_L - V_{mfi})}. \quad (31)$$

Eq. (31) is a coupled ordinary differential equation that describes the volume fraction variation of solids 1 and 2 in axial direction of the fluidised bed. However, in order to apply Eq. (31) correctly, the mass conservation of each of the solids species must be applied. Supposing that solid 1 is the lower settling velocity solid phase either because it has a smaller diameter or is less dense than the solid phase 2, then solid 1 will occupy the top portion of the fluidised bed from a vertical height $z=z_{S1,bottom}$ to $z=z_{bed}$. Similarly, the heavier solid species 2, will occupy the bottom part of the fluidised bed from $z=0$ to $z=z_{S2,upper}$. A mass balance is obtained by integrating solid mass over these vertical heights of the bed and written as:

$$M_{Si} = \begin{cases} \int_{z_{1,bottom}}^{z_{bed}} \epsilon_{Si} \rho_{Si} A dz; & \text{for } i=1 \\ \int_0^{z_{2,upper}} \epsilon_{Si} \rho_{Si} A dz; & \text{for } i=2 \end{cases}, \quad (32)$$

where M_{S1} and M_{S2} are the mass of solid species 1 and 2, respectively.

For known mass M_{S1} and M_{S2} , Eq. (31) can be solved simultaneously with Eq. (31) to obtain ϵ_{S1} , ϵ_{S2} , Z_{bed} , $Z_{1,bottom}$, $Z_{2,upper}$. A fourth order Runge-Kutta scheme was utilised to integrate Eq. (31) with the following initial conditions:

$$\text{Species 1 (lighter phase): } \epsilon_{S1}(z=0) = 0$$

Species 2 (heavier phase): $\epsilon_{S2}(z=0) = \text{Richardson-Zaki [52] for single component bed only}$. In the simulation the vertical height was incremented from $z = 0$ using intervals of $\Delta z = (d_{S1} + d_{S2})/4$ up to $z = z_{bed}$ which was determined by satisfying the mass balance given in Eq. (32).

5. Results and discussion

5.1. Comparison of present dispersion model with available models

A comparison of the present dispersion coefficient model with the models available in the literature [9, 34, 37-39, 42, 43, 45, 46], as well as numerical (CFD-DEM) predictions [20], is presented in Fig. 2. The operating and geometrical conditions were chosen to be those of Galvin et al. [9] (see Table 1), wherein the binary particle phases were fluidised with water of different superficial velocity ranges from $V_L = 0.031 \text{ m s}^{-1}$ to 0.058 m s^{-1} . The particle phases are defined such that $V_{S\infty 1} < V_{S\infty 2}$, whereby particle species 2 is denser than particle species 1.

Fig. 2a and b show the variation of the dispersion coefficient as a function of liquid superficial velocity for the lighter and denser phase particle species, respectively. The assigned values of the adjustable parameters (see Table 2) used in the correlations are $K_1 = 5.0$, $K_2 = 5.5$ [37]; $K_3 = 6.8$, $K_4 = 2.195$ [43]; and $K_5 = 0.7$ [9] whereas the liquid volume fraction was calculated using the Richardson and Zaki correlation [52]. In most of the cases, including the present model, it can be seen that the dispersion coefficient increases with increasing liquid superficial velocity due to its non-linear dependency on the velocity. However, the model predictions of Joshi et al. [39] and Bhattacharyya and Dutta [46] indicate almost

insignificant change in dispersion behaviour with superficial liquid velocity. Although the model of Joshi et al. [42] includes a linear dependency on the velocity it also includes an inverse relation to the liquid volume fraction (bed voidage), which also increases with superficial velocity and somewhat nullifies the effect of velocity. All models predict higher particle dispersion coefficient for the lighter phase particle. Physically this means that lighter solids have a lower terminal settling velocity and a higher movement and number of collisions with neighbouring particles compared with the denser phase particles. For the lighter phase particle species, Joshi et al. [39], Bhattacharyya and Dutta [46] and Murli et al. [45] all under-predict, whilst the rest of the models over-predict the dispersion coefficient compared with the present model. However, at a lower velocity (0.031 m s^{-1}), the van der Meer et al. [42], Dorgelo et al. [34], and Galvin et al. [9] predictions are very close to the present model. The Dorgelo et al. [34] model predicts higher value of dispersion coefficient with a liquid superficial velocity greater than 0.042 m s^{-1} . From Fig. 2b, it can be seen that, up to a certain velocity range (0.031 to 0.049 m s^{-1}), the present model agrees with the models of van der Meer et al. [42], Dorgelo et al. [34], Batchelor [37], Galvin et al. [9] and Murli et al. [45].

5.2. *Bed expansion behaviour (1D model prediction vs measurement)*

The 1D convective-diffusive model firstly was used to predict the expanded bed height of the binary SLFB. The operating and geometrical conditions were chosen to be those of Galvin et al. [9] and Chavan and Joshi [5] from Table 1. The particle phases 1 and 2 represent lighter/smaller and denser/larger particles, respectively. In the work of Galvin et al. [9] the binary particles had the same diameter and differed only by density; and were fluidised by water at different liquid superficial velocities. In the work of Chavan and Joshi [5] three different binary particle mixtures were used which differed in diameter but had the same density; and were fluidised by water at a fixed superficial velocity. A comparison of the model predicted bed expansion behaviour of the binary particle system in these two cases is presented in Fig. 3.

The steady state bed heights indicate the location where the free interface of the lighter phase is present corresponding to the last available sampling point. The reported experimental data indicates an increasing power law ($R^2 \sim 0.94$ (a) and $R^2 \sim 0.86$ (b)) trend in bed the height changing from (a) ~ 0.35 to 1.3 m with increasing liquid superficial velocity from 0.031 m s^{-1} to 0.058 m s^{-1} and (b) ~ 0.63 to 0.72 m with decreasing diameter ratio ranging from 2.0 to 1.17 at constant liquid superficial velocity, $V_L = 0.007 \text{ m s}^{-1}$. The numerical prediction of velocity-volume fraction relationship was validated with the experimental data which indicates good agreement with only $\sim 6\%$ deviation.

5.3. Binary solids concentration along the axial direction

The model predicted concentration profiles for binary solid phases are compared in Figs. 4-6 with the experimental measurements of Galvin et al. [9], Chavan and Joshi [5] and Juma and Richardson [10], respectively (see Table 1). The binary solid dispersion correlations by Asif and Petersen [43], Galvin et al. [9] and Murli et al. [45] reported in the earlier studies were also used in the present numerical modelling approach to investigate their predictability under different operating conditions [5, 9, 10]. Fig. 4a shows the concentration distribution of the binary (lighter and denser phase) solid along the axial direction at the liquid superficial velocity $V_L = 0.031 \text{ m s}^{-1}$ and is compared with the experimental data [9]. It can be seen that the denser phase volume fraction remained almost unchanged at ~ 0.27 up to ~ 0.12 m height from bed bottom; and a similar trend is also observed in the lighter phase where the volume fraction remains almost constant at ~ 0.22 when the bed changes from ~ 0.17 to 0.38 m. The resultant profile clearly shows the segregation behaviour in the bed which is attributed to the density difference (1600 and 1900 kg m^{-3}) between the two solid phases.

In addition to the two segregation zones (top and bottom), a small partially intermixed transition zone of height ~ 0.125 m was also observed. It can be seen that the model of Asif and Petersen [43] cannot be used to predict this mixing zone (~ 0.1 to ~ 0.2 m) height. The model of

Murli et al. [45] also under-predicted the mixing zone height as well as the overall bed expansion. The model of Galvin et al. [9]’s predicts these behaviours reasonably well. The best prediction, however, was obtained from the present model. Similar observations can be identified in Figs. 4b to 4f for increasing liquid superficial velocity cases where the model of Asif and Petersen [43] was incapable of describing the intermixing zone of two solid phases. Furthermore, at higher liquid superficial velocity (0.058 m s^{-1}), all the other dispersion model except the present model slightly over-predicted the transition zone within 12 %.

At the free interface of lighter phase, a distinct plateau can be observed in the model prediction which indicates zero concentration of lighter phase above the interface. The location of this interface was obtained by satisfying mass conservation of the lighter phase. Similar trends can be observed in Figs. 4b to 4f for the increasing liquid superficial velocity cases where good agreement between the model predictions and the experimental measurements were also obtained.

With increasing liquid superficial velocity, the concentration of the both solid phases decrease due to an increase in bed voidage (liquid volume fraction), and there is an apparent narrowing of the profiles in the horizontal direction. At the same instance, bed height expands indicating total solid mass (area under the curve) in each case remains conserved. Fig. 4a-f show that volume fraction of the lighter particle phase at the top section is less than that of the denser particle phase at the bottom section. This explains the apparently higher value of the mean free path for the lighter phase in the bottom section and the denser phase in the top section compared with that in the corresponding segregated zones. For this reason, the dispersion coefficient of the lighter phase (solid species 1) at the lower limiting position of lighter solid phase was observed to be higher compared to heavier phase (solid species 2); a similar observation was noted at the upper limiting position of denser solid phase.

The obtained RMSD for the solid phases 1 and 2 for these cases (Fig. 4) were – Eq. (8): 0.054 and 0.038, Eq. (11): 0.042 and 0.030, Eq. (12): 0.108 and 0.043, Eq. (16): 0.022 and 0.030 (Fig. 4a); Eq. (8): 0.032 and 0.026, Eq. (11): 0.066 and 0.027, Eq. (12): 0.065 and 0.031, Eq. (16): 0.022 and 0.009 (Fig. 4b); Eq. (8): 0.044 and 0.033, Eq. (11): 0.043 and 0.033, Eq. (12): 0.048 and 0.045, Eq. (16): 0.016 and 0.019 (Fig. 4c); Eq. (8): 0.023 and 0.017, Eq. (11): 0.028 and 0.023, Eq. (12): 0.023 and 0.018, Eq. (16): 0.025 and 0.011 (Fig. 4d); Eq. (8): 0.019 and 0.023, Eq. (11): 0.019 and 0.018, Eq. (12): 0.017 and 0.014, Eq. (16): 0.019 and 0.010 (Fig. 4e); Eq. (8): 0.021 and 0.028, Eq. (11): 0.019 and 0.023, Eq. (12): 0.018 and 0.027, Eq. (16): 0.014 and 0.005 (Fig. 4f), respectively.

Fig. 5 compares model predictions of the volume fraction distribution of a binary particle system along the axial direction with the experimental data of Chavan and Joshi [5]. Three different binary mixture systems differing only in particle diameter were modelled: (a) complete segregation (CS) - 0.00047 and 0.00094 m; (b) partial segregation (PS) - 0.00047 and 0.000665 m; and (c) no-segregation/complete mixing (NS) - 0.00047 and 0.00055 m, respectively. These observed bed behaviours in the equal density system was solely attributed to the difference in the diameter ratio, d_r , of these systems which governs the inertial contribution in bed. Fig. 5a shows that larger solid phase volume fraction (predicted by present model) remains almost unchanged at ~ 0.3 up to ~ 0.23 m height from bed bottom and a similar trend is also observed in the lighter phase where the volume fraction remains almost constant at ~ 0.14 when the bed height changes from ~ 0.24 to 0.64 m. A clear and stable interface can be observed in between two solid species for the largest diameter ratio case ($d_r = 2.0$).

Galvin et al. [9] and Murli et al. [45]'s models were observed to be under-predicting the stable interface, whilst Asif and Petersen [43]'s model was unsuccessful to describe the complete bed segregation pattern. When the diameter ratio was decreased to 1.41, the binary

particle species started forming an intermixing zone of height ~ 0.3 m in between the two mono-component regions (Fig. 5b). The proposed model was found to predict reasonably well both the solid phase concentration profile as well as the transition zone height. In comparison, Asif and Petersen [43]'s model under-predicts interphase as well as the length of the transition zone (~ 0.2 m). Moreover, Galvin et al. [9] and Murli et al. [45]'s dispersion models were not able to describe the partial segregation/partial mixing behaviour of the two-particle species.

Further decrease in diameter ratio to 1.17 led the bed of the binary mixture intermixed completely (Fig. 5b) throughout the SLFB. Qualitative agreement can be noted from the Asif and Petersen [46]'s model, however, the other two models were unsuccessful in describing the no-segregation/complete mixing behaviour for this diameter ratio range. The RMSD for the predicted concentration profile of solid species 1 and 2 from different dispersion models were as follow: Eq. (8): 0.015 and 0.054, Eq. (11): 0.056 and 0.092, Eq. (12): 0.055 and 0.087, Eq. (16): 0.008 and 0.004 (Fig. 5a); Eq. (8): 0.061 and 0.041, Eq. (11): 0.054 and 0.054, Eq. (12): 0.055 and 0.055, Eq. (16): 0.046 and 0.025 (Fig. 5b); Eq. (8): 0.015 and 0.054, Eq. (11): 0.056 and 0.092, Eq. (12): 0.055 and 0.087, Eq. (16): 0.006 and 0.019 (Fig. 5c).

The present convective-diffusive analysis was also applied to a SLFB system (Fig. 6) where paraffin oil (density = 840 kg m^{-3} and viscosity = $0.011 \text{ kg m}^{-1} \text{ s}^{-1}$) was used to fluidise the binary particle system [10]. It should be noted that the experimental data of the binary particle concentration distribution were limited to the transition zone [10]. From Figs. 6 a and b, it can be seen that both the proposed and Asif and Petersen [43]'s dispersion model indicate the presence of an intermixing zone, and that deviations in the predicted concentration distribution from these models were less. Galvin et al. [9] and Murli et al. [45]'s models show less accuracy in the transition region. The deviations (RMSD) in the concentration profile for the solid species 1 and 2 predicted by different dispersion models were as follow: Eq. (8): 0.016 and 0.010, Eq. (11): 0.020 and 0.026, Eq. (12): 0.034 and 0.039, Eq. (16): 0.010 and 0.008 (Fig.

5a); Eq. (8): 0.022 and 0.074, Eq. (11): 0.028 & 0.077, Eq. (12): 0.040 & 0.052, Eq. (16): 0.017 & 0.020 (Fig. 5b), respectively.

5.4. Axial variation in solid dispersion coefficient

The binary solid phase dispersion coefficient predicted from the proposed model [Eq. (16)] are plotted in Figs. 7-9 using the experimental conditions of Galvin et al. [9], Chavan and Joshi [5] and Juma and Richardson [10], respectively. Fig. 7a-f shows the axial variation in the dispersion coefficient (continuous and dotted lines correspond to heavier and lighter particle species, respectively) at increasing liquid superficial velocity ranging from $V_L = 0.031 \text{ m s}^{-1}$ to 0.058 m s^{-1} . The model predictions show that the solid dispersion coefficient varies as a function of column height due to a change in the solid volume fraction, which is consistent with the experimental observation of Galvin et al. [9]. Such variation is especially significant in the transition zone (intermixing regime between lighter and denser phase particles) while the phenomenon is almost negligible in the segregated zone comprising the mono-solid phase. It can be seen that at the lowest superficial velocity case, $V_L = 0.031 \text{ m s}^{-1}$, the local dispersion coefficient varies from 1.42×10^{-5} to $3.47 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ for solid phase 1, whilst it is 8.02×10^{-6} to $3.15 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ for solid phase 2 (Fig. 7a). For other liquid superficial velocity cases, the dispersion coefficient variations are as follows - at $V_L = 0.037 \text{ m s}^{-1}$, $D_{S1} = 2.42 \times 10^{-5}$ to $4.78 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, and $D_{S2} = 1.31 \times 10^{-5}$ to $4.60 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ (Fig. 7b); at $V_L = 0.042 \text{ m s}^{-1}$, $D_{S1} = 3.72 \times 10^{-5}$ to $6.89 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, and $D_{S2} = 2.08 \times 10^{-5}$ to $6.85 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ (Fig. 7c); at $V_L = 0.049 \text{ m s}^{-1}$, $D_{S1} = 6.93 \times 10^{-5}$ to $9.34 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, $D_{S2} = 3.20 \times 10^{-5}$ to $1.07 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ (Fig. 7d); at $V_L = 0.054 \text{ m s}^{-1}$ (Fig. 7e), $D_{S1} = 1.13 \times 10^{-4}$ to $1.30 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ and $D_{S2} = 4.61 \times 10^{-5}$ to $1.61 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$; and at $V_L = 0.058 \text{ m s}^{-1}$, $D_{S1} = 1.56 \times 10^{-4}$ to $1.97 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ and $D_{S2} = 6.24 \times 10^{-5}$ to $2.21 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ (Fig. 7f), respectively. Similar to the previously reported

models of dispersion coefficient [10, 32-34, 42], the predicted dispersion coefficient shows an increasing trend with increasing liquid superficial velocity.

Figs. 8a-c show the axial variation of binary system dispersion coefficient for decreasing solid diameter ratio in the range from $d_r = 2.0$ to 1.17 [5]. The continuous and dotted lines represent the axial variation of dispersion coefficient for larger (solid phase 2) and smaller particle (solid phase 1) phase, respectively. It can be seen that for the highest diameter ratio case ($d_r = 2.0$), which shows a complete bed segregation behaviour, the local dispersion coefficient for solid phase 1 remains almost unchanged (varies from 2.25×10^{-5} to $3.25 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$) whilst it sharply increases from 7.28×10^{-6} to $5.82 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ for solid phase 2 at the transition zone and thereafter remain almost constant (Fig. 8a). For the other d_r cases, dispersion coefficient variations are as follows: at $d_r = 1.41$ (partially segregated bed), $D_{S1} = 2.07 \times 10^{-5}$ to $3.45 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, and $D_{S2} = 8.80 \times 10^{-6}$ to $5.15 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ (Fig. 8b), and at $d_r = 1.17$ (completely mixed bed), both D_{S1} and D_{S2} vary consistently along the axial location wherein D_{S1} is in the range from 2.25×10^{-5} to $3.92 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, and D_{S2} varies from 1.26×10^{-5} to $5.26 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ (Fig. 8c), respectively.

Variation in axial dispersion coefficient particularly in the transition zone for the experimental data of Juma and Richardson [10] is given in Fig. 9 which comprises two different binary systems differing only in diameter ($d_r = 1.5$ and 2.0). The corresponding liquid superficial velocities are 0.109 and 0.093 m s^{-1} , respectively. A similar trend can be observed in the variation of dispersion coefficient in these two cases. From Fig. 9a, local dispersion coefficient varies from 4.90×10^{-4} to $6.71 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ for solid phase 1, whilst it is 2.43×10^{-4} to $1.21 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$ for solid phase 2. In Fig. 9b, D_{S1} and D_{S2} varies from 3.49×10^{-4} to $4.79 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ and 1.31×10^{-4} to $8.12 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$, respectively.

5.5. Analysis of intermixing of binary particle species

Fig. 10 shows the variation of intermixing zone height of the binary SLFB for two different cases: (a) Galvin et al. [9] and (b) Chavan and Joshi [5]. The open circle ($\circ$) data represents the lower limiting position of the first (lighter/smaller) solid phase along the axial location (present dispersion model), whilst the closed circle ($\bullet$) data presents the upper limiting position of the second (denser/larger) solid phase. These limiting values can be obtained from the model predicted binary solid volume fraction earlier shown in Figs. 4-5. The distance between the two limiting points at each liquid superficial velocity or diameter ratio represents the intermixing zone height. In Fig. 10a, it can be seen that with increase in the liquid superficial velocity the intermixing zone height also increases linearly due to increase in the liquid volume fraction. The existence of the mixed zone can be explained by the relative dominance of the convective effect over gravity effect at higher liquid superficial velocity cases which promotes dispersion of one solid phase into another. Also, it is evident from Fig. 10b that mixing zone height decreases when the diameter ratio of the binary species is increased (keeping the density ratio unity) due to dominant inertia which promotes the segregation tendency. Specifically, when the diameter ratio is increased to $d_r = 2$, the mixing zone disappears, and the bed becomes completely segregated.

6. Conclusion

The specific findings of this study are summarised below:

- A new correlation of particle dispersion coefficient for liquid fluidised beds has been

proposed: $D_s = 0.15\lambda_s \left(\frac{V_L - V_{mf}}{\epsilon_L} \right) \left(\frac{V_{S\infty}}{V_{mf}} \right)$ which is valid over the following range of the

dimensionless variables: $4 \leq Re \leq 2820$; $0.43 \leq \epsilon_L \leq 0.95$. The proposed dispersion model produces better agreement with experimental data (higher R^2 value ~ 0.7 and lower RMSD $\sim 0.0006 \text{ m}^2 \text{ s}^{-1}$) compared with the other available correlations.

- A convective-diffusive type 1D numerical model has been developed to predict variations in the concentration of each phase in the axial direction incorporating the proposed dispersion model. The numerical model was firstly validated to predict the overall SLFB expansion and the outcomes shows a reasonable agreement within 6% of deviation whilst other models underpredicted/overpredict solid concentration within 20 % deviation.
- The convective-diffusive model was also found capable of predicting different bed hydrodynamics such as complete segregation (e.g., 0.00047 and 0.00094 m binary mixture having equal density 1070 kg m^{-3}), partial segregation (e.g., 1600 and 1900 kg m^{-3} dense binary particle mixture of equal diameter 0.00109 m) and no segregation (e.g., 0.00047 m and 0.00055 m binary mixture having equal density 1070 kg m^{-3}) behaviour.
- Contrary to the common assumption that dispersion coefficient remains constant in SLFB, it was shown that when dependency on solid phase volume fraction was included, significant spatial variations of this parameter occurred specifically in the intermixing zone.
- Height of the intermixed zone was shown to increase linearly with increasing liquid superficial velocity when the two species differ by their density only. The zone height was observed to reduce significantly when the diameter ratio of particles was increased from 1.17 to 2.0 in the system.

Nomenclature

d	diameter of cylindrical bead, [m]
d_s	particle diameter, [m]
D_c	column diameter, [m]
D_e	equivalent diameter, $2\epsilon_L d_s / 3\epsilon_S$, [m]
D_{exp}	experimental value dispersion coefficient, [$\text{m}^2 \text{s}^{-1}$]
D_{Si}	axial dispersion coefficient, [$\text{m}^2 \text{s}^{-1}$]
D_{model}	model prediction of dispersion coefficient, [$\text{m}^2 \text{s}^{-1}$]

F_r	Froude number, [-]
g	gravitational acceleration, [m s^{-2}]
h_e	expanded bed height, [m]
K_1	constant, varies from 0.5 to 5.0, [-]
K_2	constant, equal to 5.5, [-]
K_3	constant, equal to 7.9 ± 1.1 , [-]
K_4	constant, equal to 2.141 ± 0.054 , [-]
K_5	constant, varies from 0.7 to 7, [-]
K_6	constant, is equal to 2.0, [-]
l	length of cylindrical bead, [m]
l_z	location of the pressure sensor, [m]
M_p	mixing efficiency, [-]
M_s	solid mass, [g]
n	Richardson-Zaki index, [-]
n_0	number of particles in a compartment at the threshold of relaxation, [-]
n_∞	number of particles in a compartment at a new steady fluidised state, [-]
N	number of data point of dispersion coefficient, [-]
Pe	Peclet number, $Re.Sc$ [-]
Re	Reynolds number, $D_c V_L \rho_L / \mu_L$, [-]
Re_m	modified Reynolds number including voidage, $d_s V_L / \mu_L \epsilon_L$, [-]
Sc	Schmidt number, $\mu_L / \rho_L D_s$, [-]
V_L	liquid superficial velocity, [m s^{-1}]
V_{pi}	classification velocity, [m s^{-1}]
V_s	interstitial velocity, V_L / ϵ_L , [m s^{-1}]
$V_{S\infty}$	terminal settling velocity of particle, [m s^{-1}]

V_{mf} minimum fluidisation velocity, [$m\ s^{-1}$]

V_L liquid superficial velocity, [$m\ s^{-1}$]

z axial length, [m]

Greek letters

ϵ_L bed voidage, [-]

ϵ_S solid volume fraction, [-]

ϵ_{nSi} normalised solid volume fraction, [-]

λ_S Mean free path, [m]

μ_L viscosity of liquid, [$kg\ m^{-1}\ s^{-1}$]

μ_S solid viscosity, [$kg\ m^{-1}\ s^{-1}$]

μ_T turbulent viscosity, [$kg\ m^{-1}\ s^{-1}$]

ρ_L liquid density, [$kg\ m^{-3}$]

ρ_S solid density, [$kg\ m^{-3}$]

ρ_{SM} solid mixture density along the axial direction, [$kg\ m^{-3}$]

τ_i travelling time for an individual particle, [s]

$\bar{\tau}$ mean value of τ_i , [s]

Subscripts

1 solid particle 1 (smaller/lighter)

2 solid particle 2 (larger/denser)

c column

e equivalent

exp experiment

L liquid phase

m modified

mf at minimum fluidisation

π	classification
S	solid phase
∞	infinite medium

Abbreviations

3D	three dimensional
CS	complete segregation
MFP	mean free path
NS	no segregation/complete mixing
PS	partial segregation
RMSD	root mean square deviation
SLFB	solid-liquid fluidised bed

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List of Tables

Table 1. Theoretical models and experimental techniques used to quantify dispersion coefficient.

Investigat ors	Model	Measurem ent technique	Experimental conditions							
			Fluidising conditions	SLF B type	H m	Dc m	ds m	ρs kg m ⁻³	ρL kg m ⁻³	μL kg m ⁻¹ s ⁻¹
Muchi et al. [31]	Diffusio n model	Pulse Response	Steady/unste ady	Mon o	0.5 3	0.07 1	0.0006 8 0.0008 4	271 0 236 0	100 0	0.001
Handly et al. [32]	Correlati on function	Pursuit of single particle motion	Steady	Mon o	1.0	0.07 5	0.0010 9	250 0	867	0.000 6
Kennedy and Bretton [11]	Diffusio n model	Concentrati on profile, classificatio n flux balances dispersion flux, two bed technique	Steady	Binar y	0.5 8	0.02 5	0.002 0.001	247 0 286 2	100 0	0.001
Carlos and Richardson [33]	Diffusio n model	Step response	Unsteady	Mon o	1.2 2	0.10 2	0.009	249 0	119 0	0.010
Al-Dibouni and Garside [47]	Diffusio n model	Concentrati on profile, classificatio n flux balances dispersion flux, two bed technique	Steady	Multi	1.4	0.05 2	0.0030 7 0.0026 0.0022 0.0018 3	230 0	100 0	0.001
Juma and Richardson [10]	Diffusio n model	Concentrati on profile, classificatio n flux balances dispersion flux, two bed technique	Steady	Binar y	1.9	0.10 5	0.002 0.003 0.004	296 0	840 100 0	0.011 0.001
van der Meer et al. [42]	Diffusio n model	Concentrati on profile, classificatio n flux balances dispersion flux, two bed technique	Steady	Binar y	1.0	0.06 1 0.05 1 0.04 3 0.02 1	0.0006 7	132 0 137 5	100 0	0.001

Investigat ors	Model	Measurem ent technique	Experimental conditions							
			Fluidising conditions	SLF B type	H m	Dc m	ds m	ρ_s kg m ⁻³	ρ_L kg m ⁻³	μ_L kg m ⁻¹ s ⁻¹
Dorgelo et al. [34]	Stochasti c model	Random walk method	Steady	Mon o	4.3	0.23 0	0.023	245 5	115 4	0.010
Yutani et al. [12, 35]	Stochasti c model	Relaxation method	Unsteady	Mon o	0.7	0.05 4	0.0004 2 0.0005 4 0.0007 7	250 0	100 0	0.001
Kang et al. [38]	Stochasti c model	Relaxation method	Steady or unsteady	Mon o	2.5	0.15 2	0.001 0.003 0.006	250 0	100 0	0.001
Barghi et al. [4]	Diffusio n model	Collision method	Steady	Binar y	2.7 1	0.17 2	0.003 0.005	255 0	100 0	0.000 9
Galvin et al. [9]	Diffusio n model	Concentrati on profile, classificatio n flux balances dispersion flux	Steady	Binar y	2.0	0.05	0.001	160 0 190 0	100 0	0.001
Murli et al. [45]	Diffusio n model	Concentrati on profile, classificatio n flux balances dispersion flux	Steady	Multi	1.6	0.15	0.0009 95 0.0008 51 0.0007 09 0.0006 0.0005 02 0.0003 89	107 5	100 0	0.001
Chavan and Joshi [5]	Diffusio n model	Concentrati on profile, classificatio n flux balances dispersion flux	Steady	Binar y	1.2	0.05	0.0047 0.0055 0.0067 0.0094	107 0	100 0	0.001

Table 2. Proposed correlations for dispersion coefficient.

Investigator	Solid type	Correlation	Eq.
Muchi et al. [31]	Mono	$D_i = 0.04\rho_{SM}\mu_L \left(\text{Re}_m / 1 - \epsilon_L \right)^{1.7}$	(1)
Kennedy and Bretton [11]	Binary	$D_i (\partial \epsilon_{Si} / \partial z) = - \epsilon_{Si} V_{pi}$	(2)
van der Meer et al. [42]	Binary	$D_i = 0.25V_L^{2.2}$; when $0.5 < \epsilon_L < 0.9$ and $0.002 < V_L < 0.30$	(3)
Dorgelo et al. [34]	Mono	$D_i = 0.1V_L^2$	(4)
Yutani et al. [35]	Mono	$D_i = (1/2\bar{\tau}) l_z^2 (n_\infty / n_o) (1 - n_\infty / n_o)$	(5)
Batchelor [37]	Mono	$D_i = (K_1 d_{Si} V_L \epsilon_L^{K_2}) / 2$; $K_1 \sim 0.5$ to 5.0 , $K_2 = 5.5$	(6)
Kang et al. [38]	Mono	$D_i = 2.97 \times 10^{-3} (V_L + V_{mf_i})^{0.802}$	(7)
Asif and Petersen [43]	Binary	$D_i = (K_3 g d_{Si}^2 / V_L \epsilon_L) (V_L - V_{mf_i} / V_{s\infty_i})^{K_4} (\rho_{Si} - \rho_L / \rho_L)$ $K_3 = 7.9 \pm 1.1$, $K_4 = 2.141 \pm 0.054$	(8)
Joshi et al. [39]	Mono	$D_i = 3 [(1 - \epsilon_L) / \epsilon_L] V_L d_{Si}$	(9)
Bhattacharyya and Dutta [46]	Binary	$D_i = \frac{\mu_L}{\rho_L} \left[\left(\frac{0.02438 d_{Si} g}{\epsilon_{i,eff} V_L^2} \right) \left(\frac{V_L - V_{mf_i}}{V_{s\infty_i} - V_L} \right)^{0.9209} \right]^{-2}$	(10)
Galvin et al. [9]	Binary	$D_i = [K_5 d_{Si} V_L / \{\epsilon_L (1 - \epsilon_L)\}]$; $K_5 = 0.7$ to 7 .	(11)
Murli et al. [45]	Multi	$D_i = 2.17 D_e \left\{ (V_L - V_{mf_i}) / \epsilon_L \right\}$.	(12)

Table 3. R^2 value of the different dispersion correlations.

No	Investigator	RMSD ($\text{m}^2 \text{s}^{-1}$)	$\ln(D_{\text{exp}}/D_{\text{model}}) = A$	
			A	R^2
1.	Muchi et al. [31]	11.25	0.75	0.54
2.	van der Meer et al. [42]	0.004	1.13	0.66
3.	Dorgelo et al. [34]	0.002	1.10	0.74
4.	Batchelor [37]	0.003	1.50	0.70
5.	Kang et al. [38]	0.004	0.95	0.31
6.	Asif and Petersen [43]	0.002	0.95	0.64
7.	Joshi et al. [39]	0.004	1.13	0.69
8.	Bhattacharyya and Dutta [46]	0.003	1.24	0.65
9.	Galvin et al. [9]	0.087	1.05	0.47
10.	Murli et al. [45]	0.022	1.04	0.52

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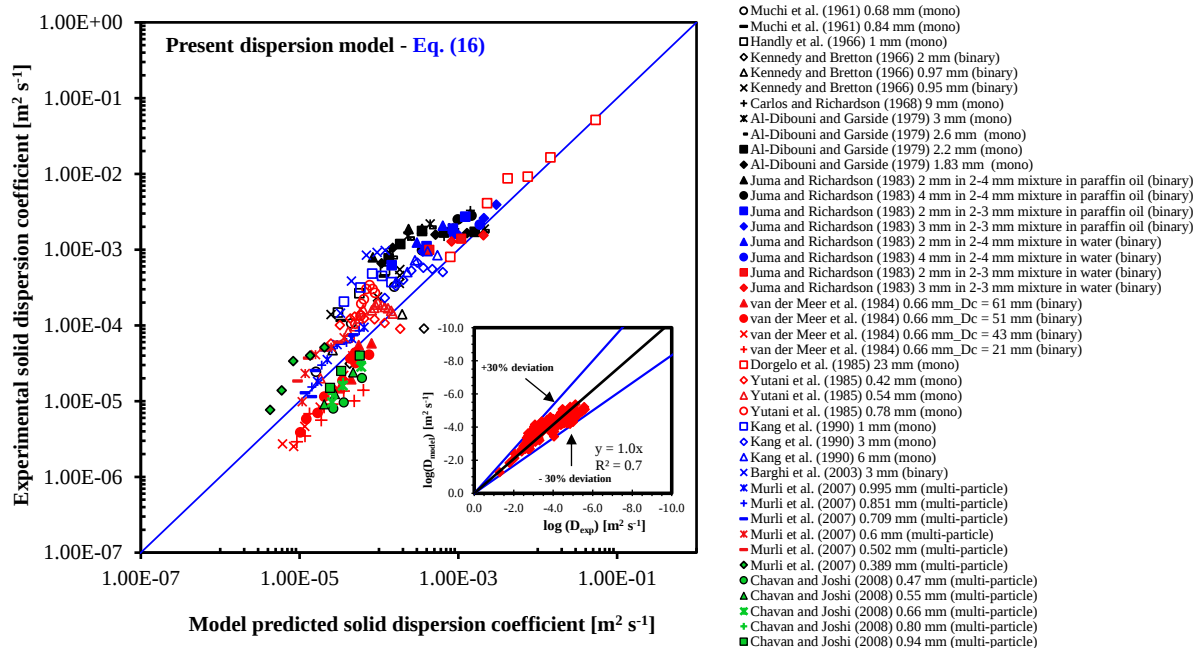


Fig. 1. Parity plot of the proposed model.

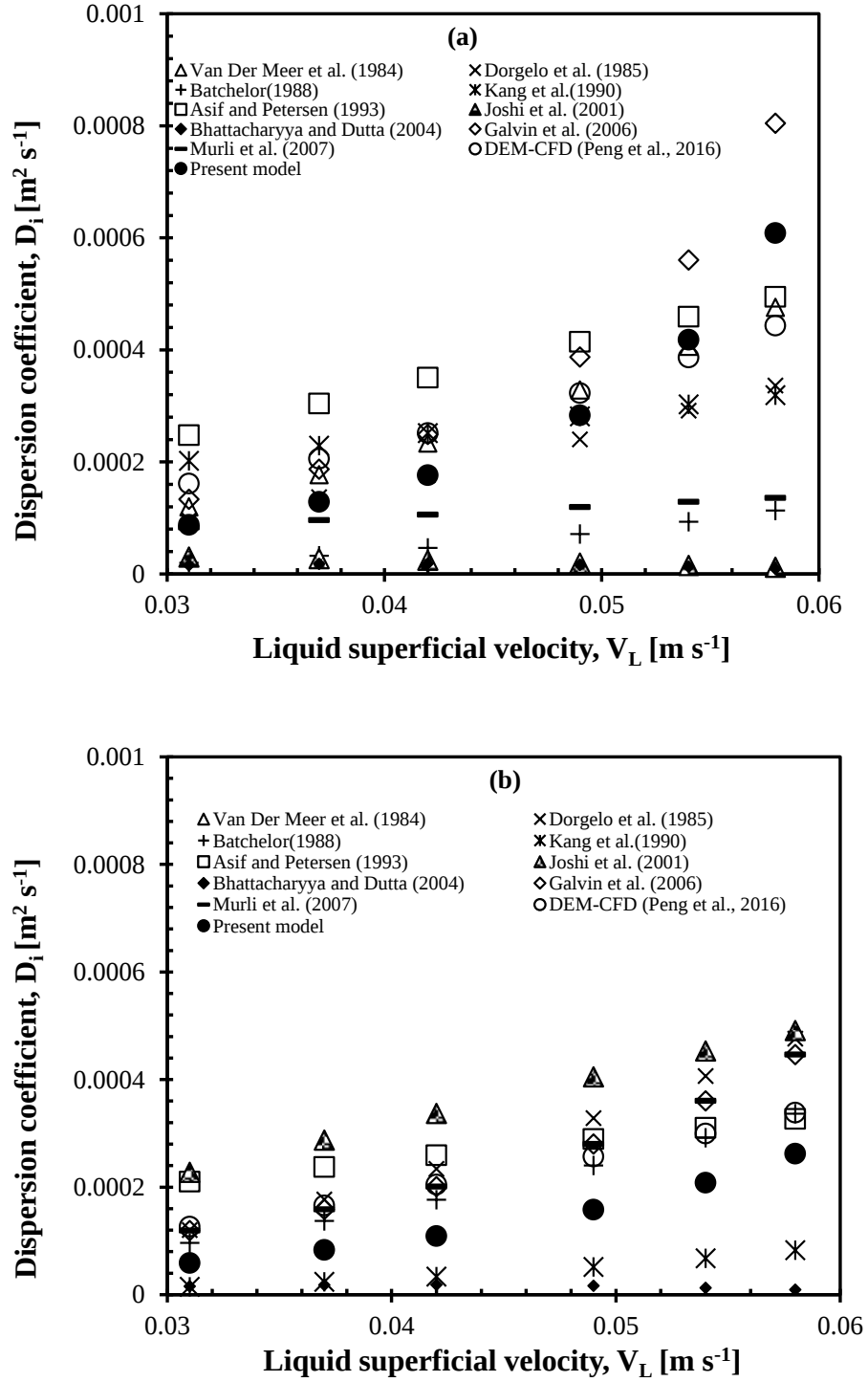


Fig. 2. Variation of dispersion coefficient as a function of liquid superficial velocity for (a) lighter phase particle species ($\rho_{s1} = 1600 \text{ kg m}^{-3}$) and (b) denser phase particle species ($\rho_{s2} = 1900 \text{ kg m}^{-3}$).

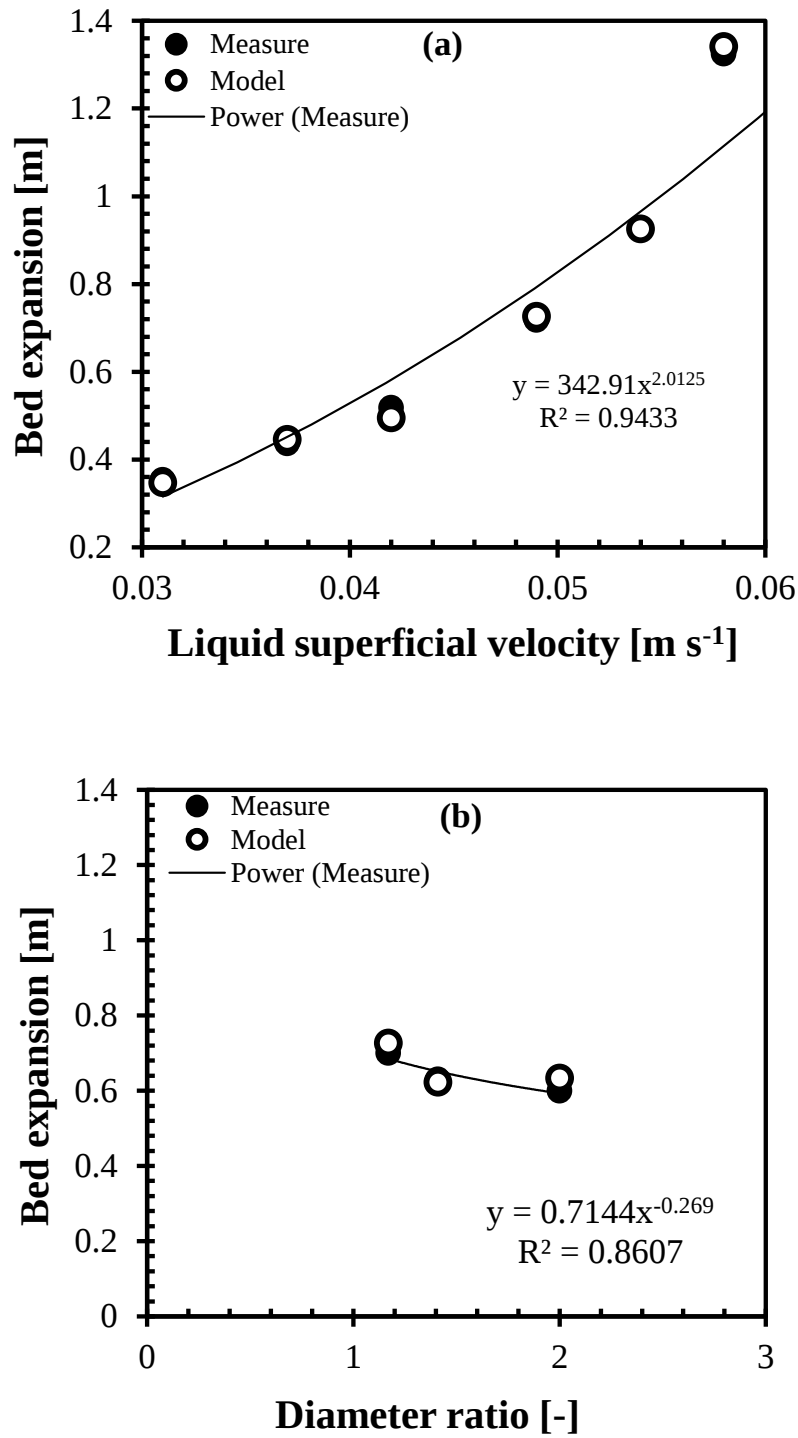


Fig. 3. Bed expansion vs liquid superficial velocity and diameter ratio (validation of numerical model): (a) Galvin et al. [9] and (b) Chavan and Joshi [5].

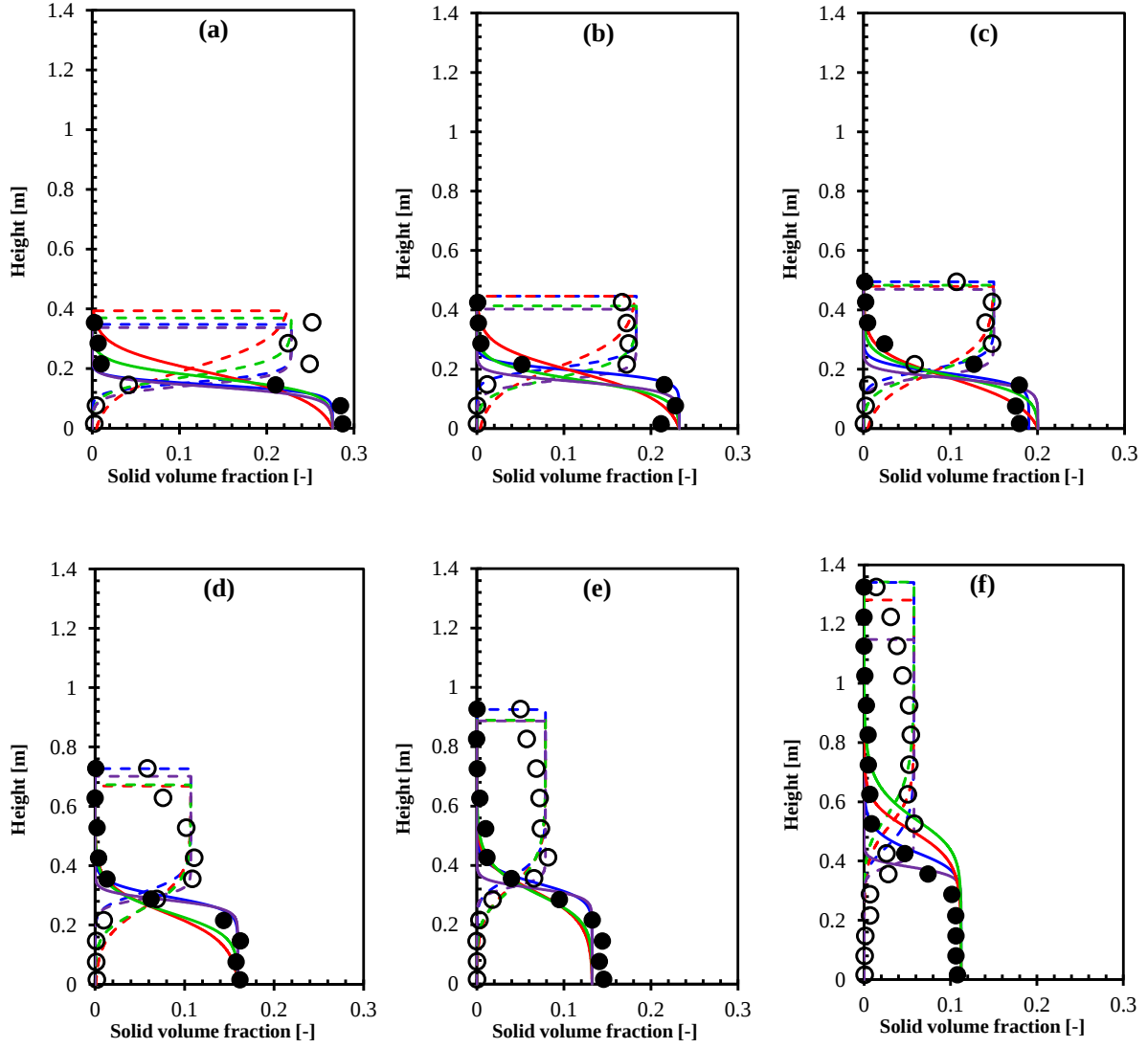


Fig. 4. Solid volume fraction vs axial height for 1600 and 1900 kg m^{-3} binary particle mixture of 1 mm diameter at different liquid superficial velocity (a) 0.031 m s^{-1} , (b) 0.037 m s^{-1} , (c) 0.042 m s^{-1} , (d) 0.049 m s^{-1} , (e) 0.054 m s^{-1} , (f) 0.058 m s^{-1} : Solid 1: $\circ$ experiment, dispersion model used in Eq. (31): --- Eq. (8), --- Eq. (11), --- Eq. (12), --- Eq. (16); Solid 2: $\bullet$ experiment, dispersion model used in Eq. (31): — Eq. (8), — Eq. (11), — Eq. (12), — Eq. (16).

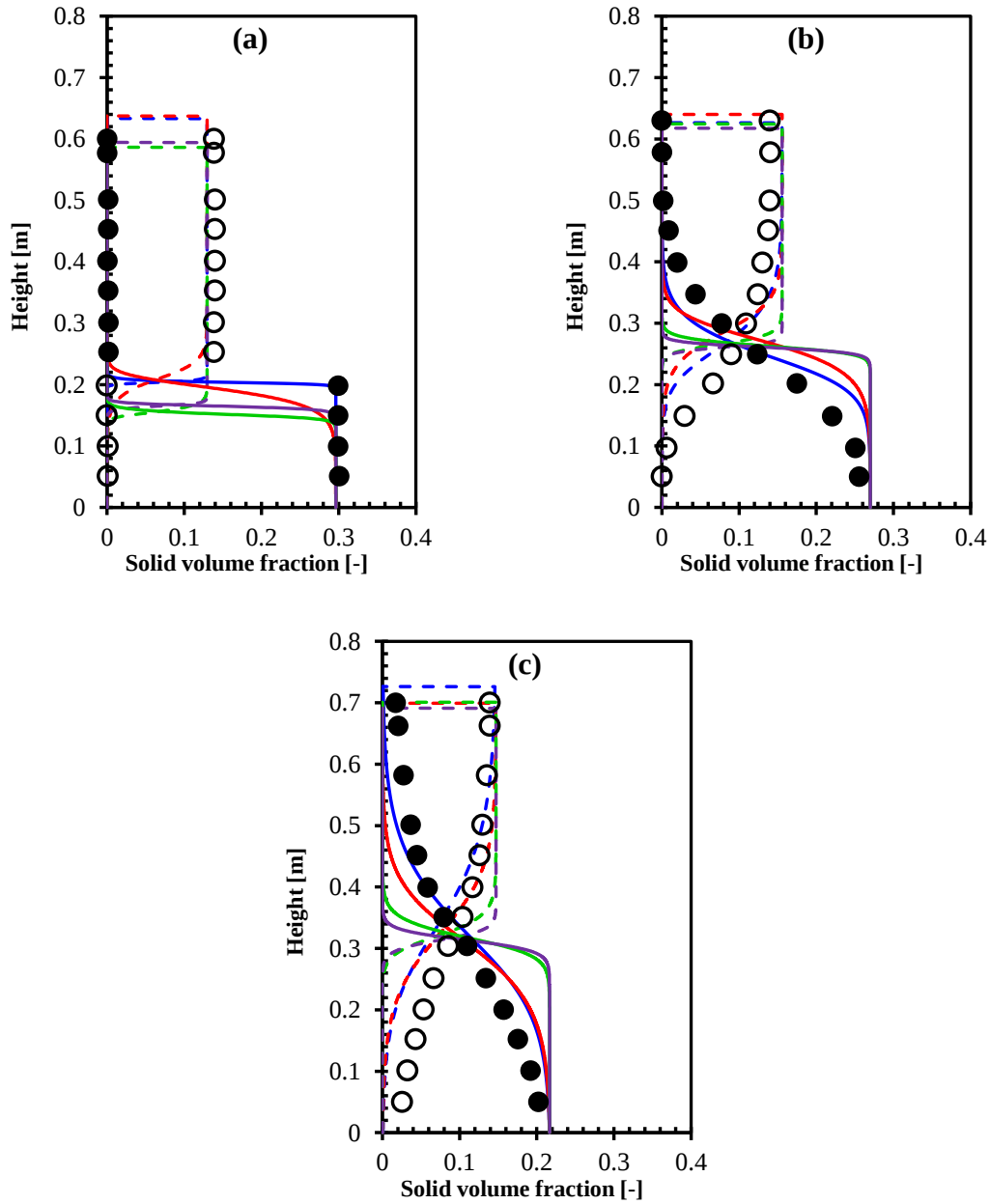


Fig. 5. Solid volume fraction vs axial height at $V_L = 0.007 \text{ m s}^{-1}$ for different binary mixture having equal density (1070 kg m^{-3}) (a) 0.00047 & 0.00094 m; (b) 0.00047 & 0.000665 m; (c) 0.00047 & 0.00055 m: Solid 1: $\circ$ experiment, --- Eq. (8), --- Eq. (11), --- Eq. (12), --- Eq. (16); Solid 2: $\bullet$ experiment, — Eq. (8), — Eq. (11), — Eq. (12), — Eq. (16).

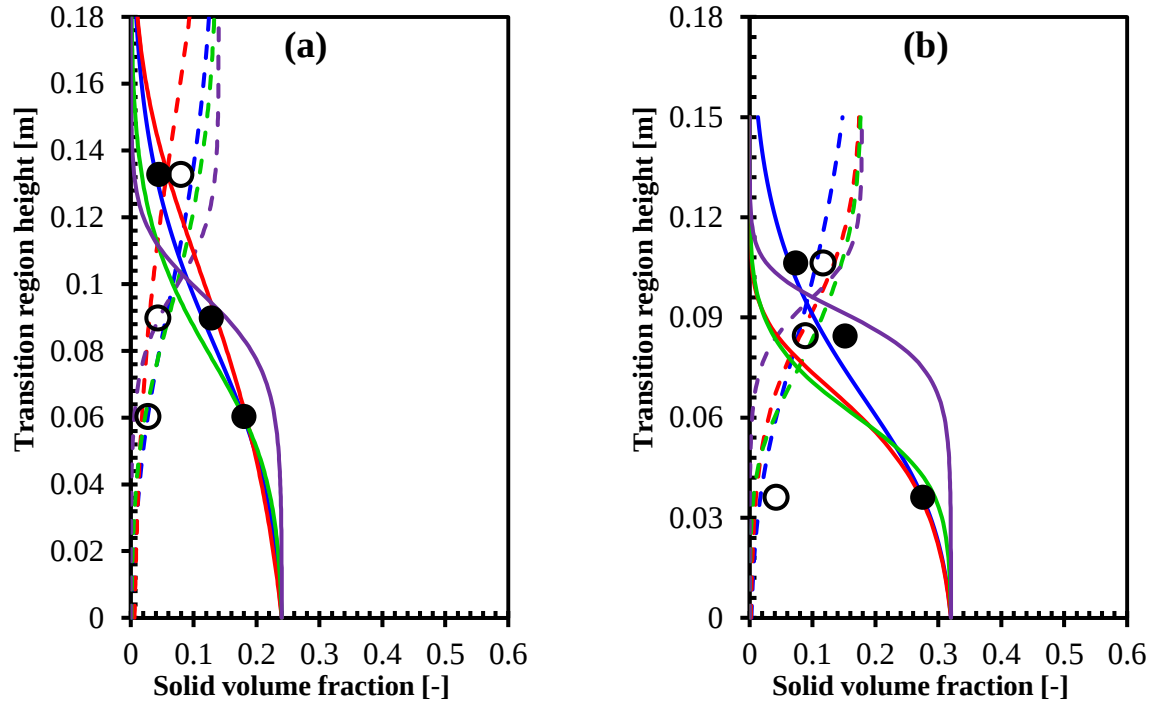


Fig. 6. Solid volume fraction vs transition region height for different binary mixture of equal density (2960 kg m^{-3}): (a) 0.002 and 0.003 m (at $V_L = 0.109 \text{ m s}^{-1}$); (b) 0.002 and 0.004 m (at $V_L = 0.093 \text{ m s}^{-1}$). Solid 1: $\circ$ experiment, --- Eq. (8), --- Eq. (11), --- Eq. (12), --- Eq. (16); Solid 2: $\bullet$ experiment, — Eq. (8), — Eq. (11), — Eq. (12), — Eq. (16).

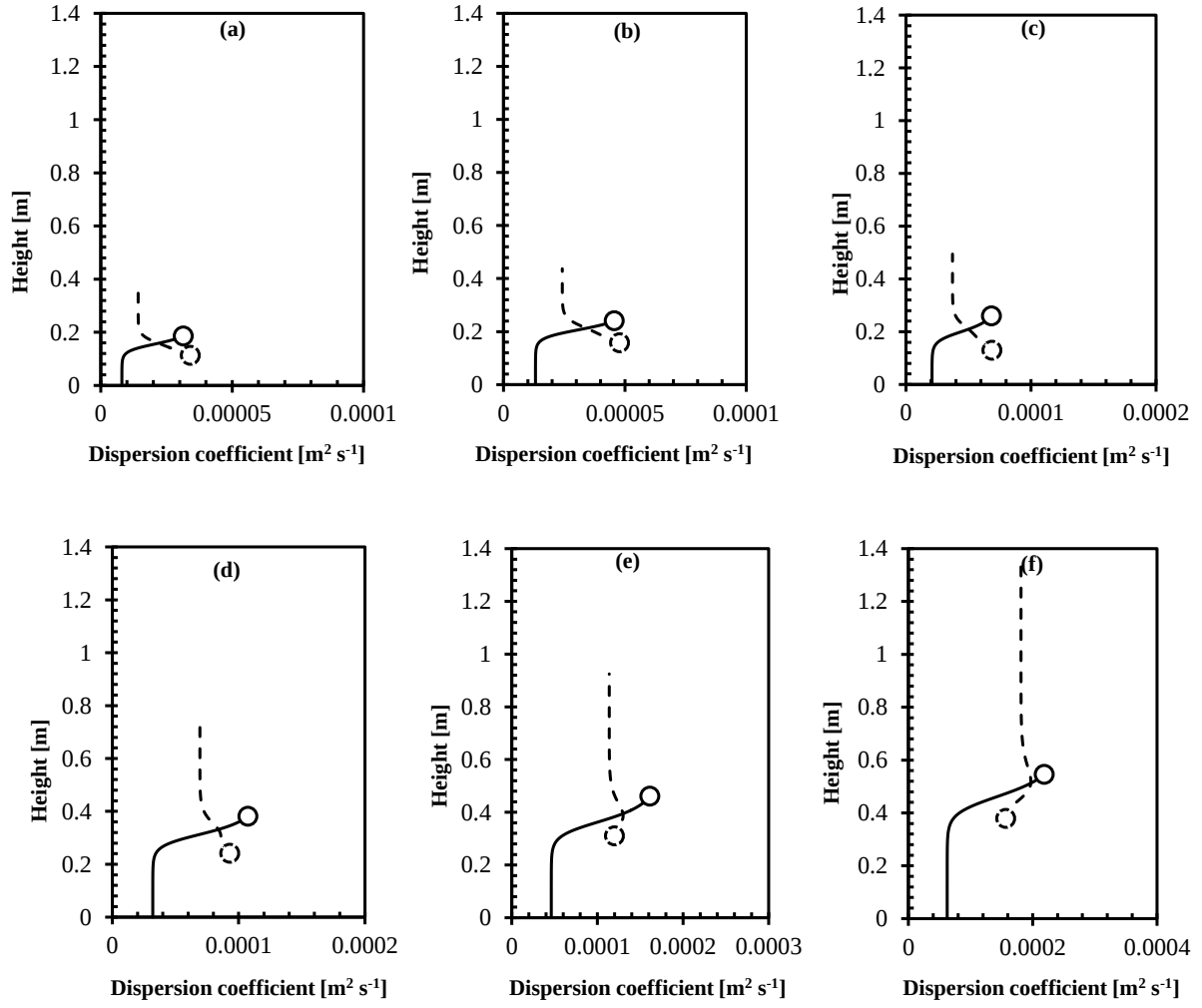


Fig. 7. Dispersion coefficient vs axial height for 1600 and 1900 kg m^{-3} binary mixture of equal diameter ($d_{s1} = d_{s2} = 0.00109 \text{ m}$) at different liquid superficial velocity, V_L , (a) 0.031 m s^{-1} , (b) 0.037 m s^{-1} , (c) 0.042 m s^{-1} , (d) 0.049 m s^{-1} , (e) 0.054 m s^{-1} , (f) 0.058 m s^{-1} : solid species 1: ---- Eq. (29), solid species 2: — Eq. (29).

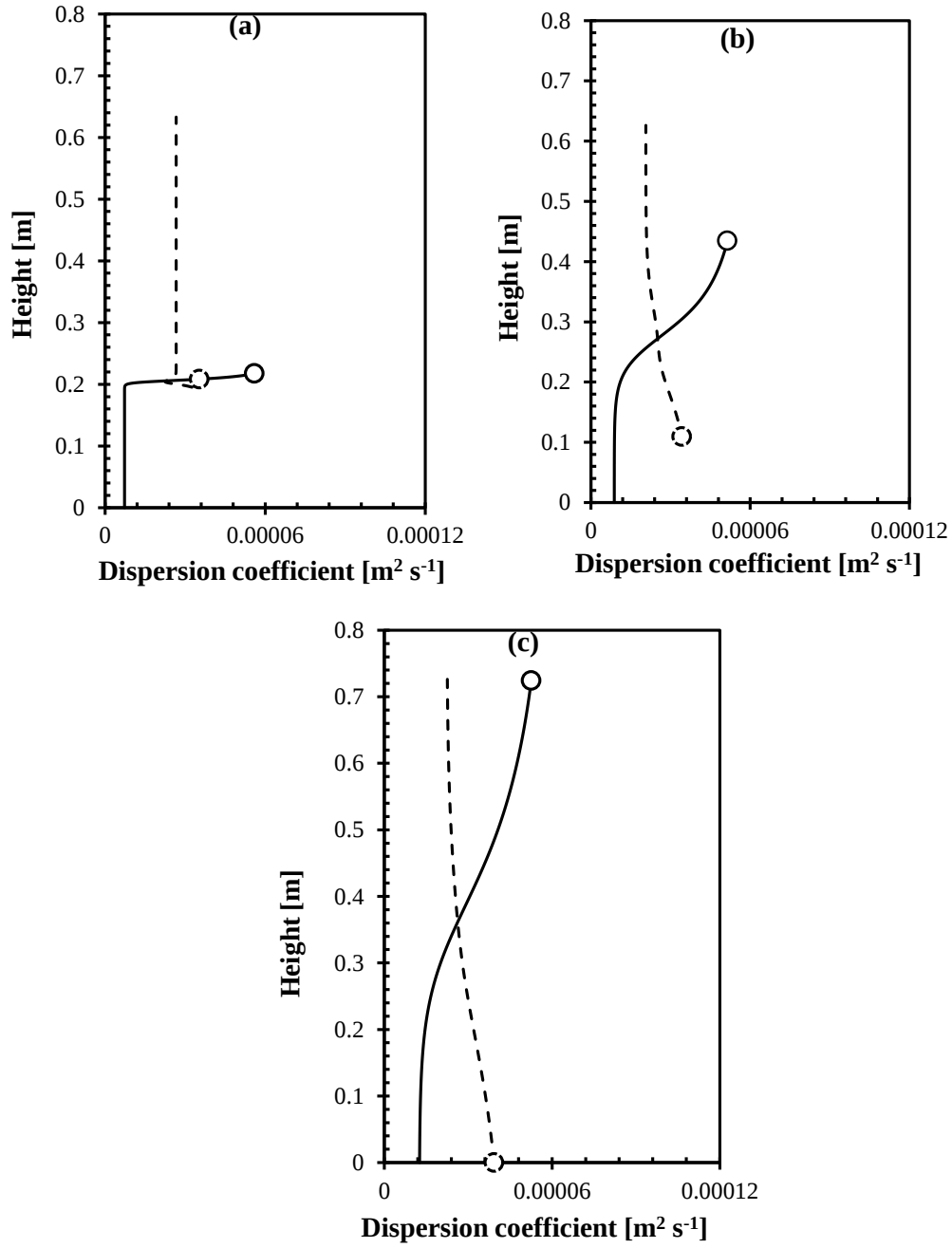


Fig. 8. Dispersion coefficient vs axial height at $V_L = 0.007 \text{ m s}^{-1}$ for different binary mixture having equal density (1070 kg m^{-3}) (a) 0.00047 and 0.00094 m ($d_r = 2.0$); (b) 0.00047 and 0.000665 m ($d_r = 1.41$); (c) 0.00047 and 0.00055 m ($d_r = 1.17$): solid species 1: ---- Eq. (29), solid species 2: — Eq. (29).

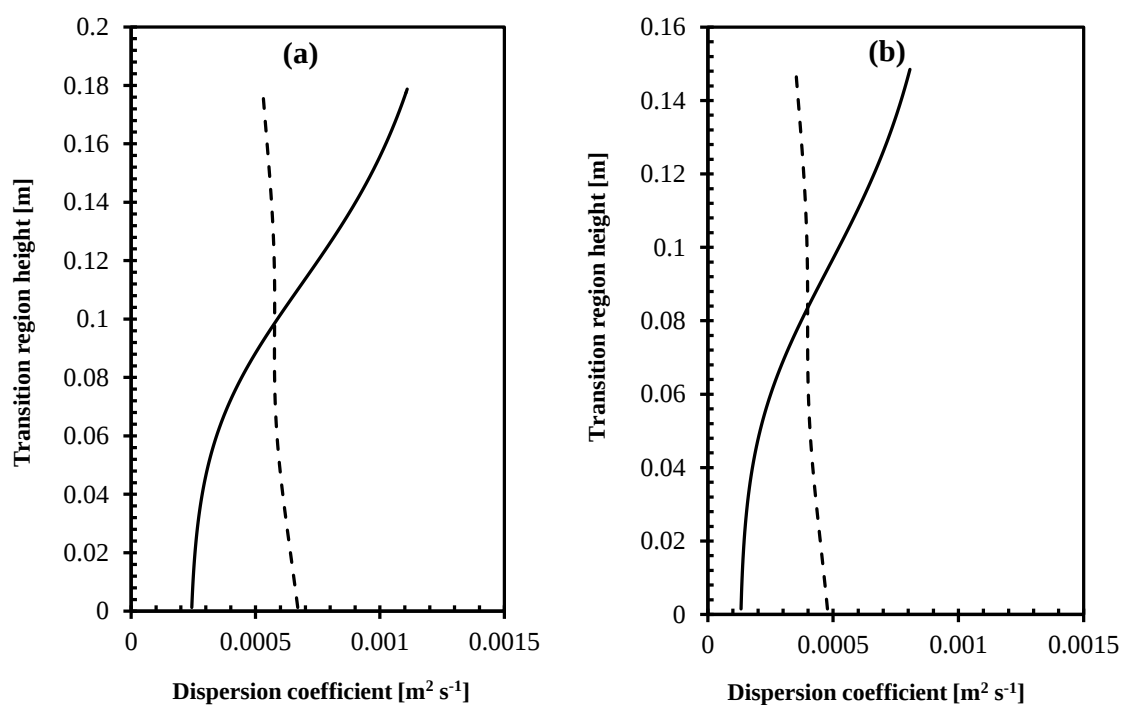


Fig. 9. Variation in dispersion coefficient in the transition region for different binary mixture with equal density (2960 kg m^{-3}) but differing in diameter: (a) 0.002 and 0.003 m (at $V_L = 0.109 \text{ m s}^{-1}$); (b) 0.002 and 0.004 m (at $V_L = 0.093 \text{ m s}^{-1}$). solid species 1: ---- Eq. (29), solid species 2: — Eq. (29).

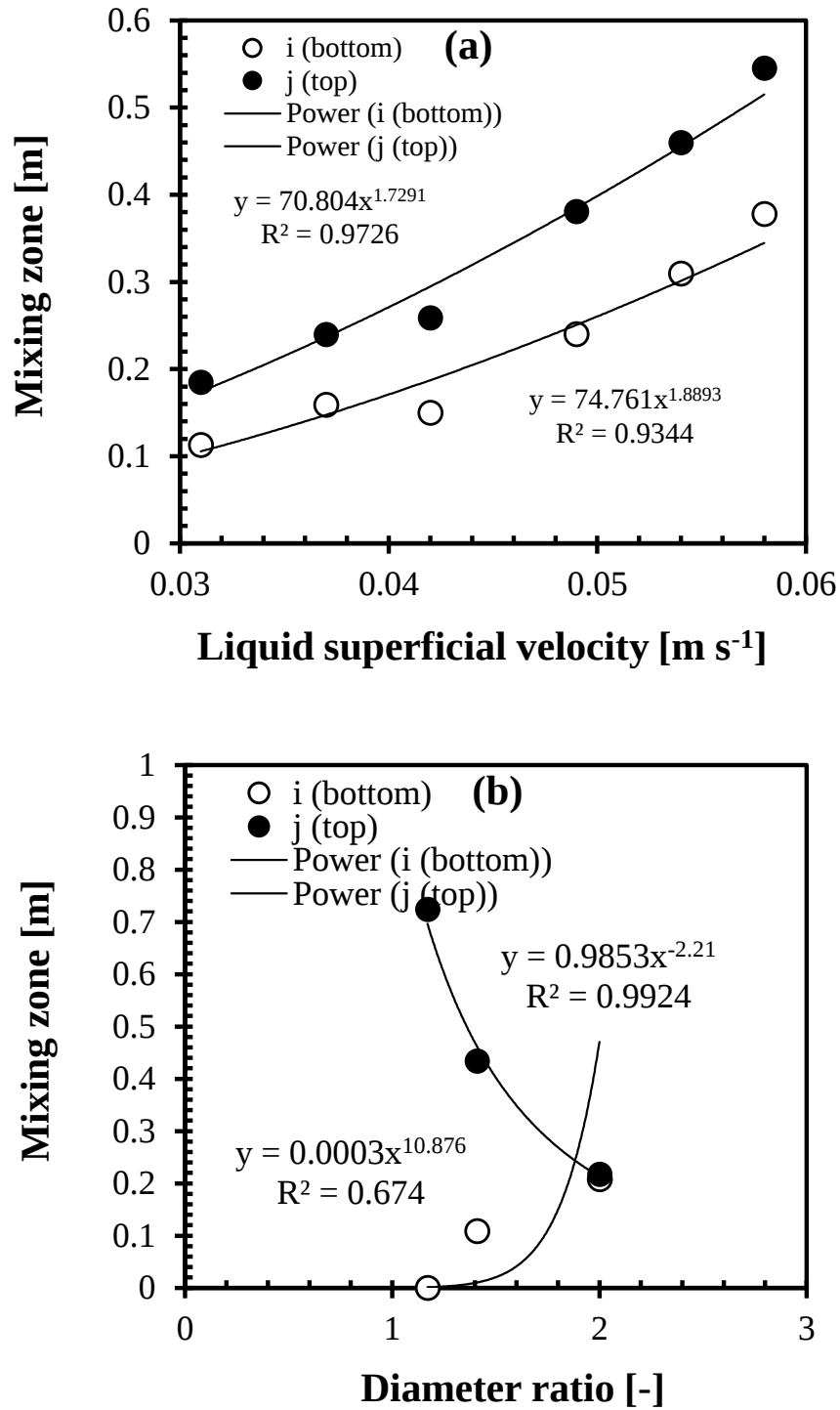


Fig. 10. Mixing zone height vs liquid superficial velocity and solid diameter ratio: (a) Galvin et al. [9] and (b) Chavan and Joshi [5].

Highlights

- Previous studies on dispersion coefficient in SLFB systems reviewed.
- A generalized correlation of dispersion coefficient is proposed.
- Compared proposed model predictions with the existing ones.
- Effect of particle diameter ratio and superficial velocity on bed behaviour studied.
- Spatial variation in particle volume fraction analysed with the model.

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